Loop Quantum Cosmology of Bianchi I Model in $\bar{\mu}$ and $\bar{\mu}'$ Schemes with Higher Order Holonomy Corrections

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The detailed formulation of loop quantum cosmology with higher order holonomy corrections has been constructed recently in the homogeneous and isotropic spacetime, yet it is important to extend the higher order holonomy corrections to include the effects of anisotropy which typically grow during the collapsing phase. In this paper we investigate the Bianchi I model in $\bar{\mu}$ and $\bar{\mu}'$ schemes respectively. First we construct the effective dynamics with higher order holonomy corrections in a massless scalar field, then we extend it to the inclusion of an arbitrary matter. Besides that, we also analyze the behavior of the anisotropy during the evolution of the universe. We find that as in the isotropic case, the singularity is never approached and the quantum bounces are generic, regardless of the order of the holonomy corrections. Some differences in the bouncing phase of the two schemes are also found out. It is also shown that in the two schemes the behavior of the anisotropy is not the same before and after the bounces.

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I. INTRODUCTION

loop quantum gravity (LQG) is a mathematically well-defined, nonperturbative, and background independent quantization of gravity [1]. The applications of LQG to homogeneous and isotropic spacetime results in loop quantum cosmology (LQC). The comprehensive formulation of LQC is constructed in the spatially flat, isotropic model in detail [2–4], which indicates that the classical big-bang singularity can be replaced by a big-bounce. With these successes, the methods can also be extended to Bianchi I model to include anisotropy [5–8].

The underlying dynamics in LQC is governed by the discreteness of the quantum geometry. Although the scheme is fascinating, the rigorous approach is difficult to afford. However, using semiclassical strategies we can also construct an effective method which has captured to a very good approximation in the quantum dynamics [9]. With quantum corrections to the classical Hamiltonian, we can get the effective equations of the modified Hamiltonian which is an efficient approach to investigate the evolution of the early universe. In [10] the evolution of the universe is investigated in the form of effective approach, which indicates that the presence of a big-bounce is a generic feature of LQC and does not require any exotic matter that violates energy condition.

When the effective approach is extended to the anisotropic Bianchi I model, the scheme is ambiguous as there are two strategies: $\bar{\mu}$ scheme and $\bar{\mu}'$ scheme, each of which is of particular interest. In [6] and [7] the $\bar{\mu}$ scheme is investigated and in [8] the $\bar{\mu}'$ scheme is considered. Not only do the results at the level of effective dynamics agree with the anticipations of the rigorous quantum approach but some details of quantum effects during the evolution are also obtained.

Despite the fascinating and attractive features, whether the quantum effects result from the discreteness of the spacetime geometry of LQC is still questionable, as some of the results through the rigorous quantum approach can also be obtained through the heuristic effective dynamics in continuum spacetime. In response to the deficiency, a new avenue of higher order holonomy corrections is investigated [11].

The approach of the higher order holonomy corrections is also a heuristic effective strategy that is more general than the conventional scheme, which takes the traditional one to be the situation where the order of the holonomy corrections is 0. In [11] it reveals that the the big-bounce are a generic feature of LQC no matter whether the higher order holonomy corrections are included and the matter density remains finite with an upper bound. In [12] the rigorous quantum theory of LQC with higher order holonomy corrections is formulated and the anticipations of [11] are confirmed. It is also shown that the higher order holonomy corrections can be interpreted as a result of admitting generic SU(2) representations for the Hamiltonian constraint operators.

The heuristic analysis of higher order holonomy corrections is a very promising approach. However, the extension to the anisotropic case is still an open issue. In this paper we investigate the higher order holonomy corrections in Bianchi I model.

Firstly we construct the effective dynamics of Bianchi I model with higher order holonomy corrections. In previous researches of $\bar{\mu}$ scheme the effective dynamics has been constructed with a massless scalar field as well as the inclusion of arbitrary matter [7]. Here we extend it to the case of higher order holonomy corrections. On the other hand, due to the complexity the detailed dynamics with arbitrary matter in $\bar{\mu}'$ scheme has not been

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completed. In this paper, by introducing a trick of series expansion we can indeed construct the effective dynamics with arbitrary matter and extend it to higher order holonomy corrections.

The anisotropy is also an important aspect as it grows in the contracting phase of the evolution. Besides that, it can also tell us some information of the universe before the big bounce. In this paper we also investigate the evolution of the the anisotropy with higher order holonomy corrections and compare the differences of the behavior in the two schemes.

This paper is organized as follows. Firstly, we review briefly the classical dynamics in Bianchi I model in Sec. II, and then, in Sec. III, we introduce the effective loop quantum dynamics with higher order holonomy corrections and two sets of research schemes: $\bar{\mu}$ scheme and $\bar{\mu}'$ scheme. Next in Sec. IV and Sec. V, we investigate in detail the effective dynamics in the forms of $\bar{\mu}$ scheme and $\bar{\mu}'$ scheme, respectively. In Sec. VI, the anisotropies of the Bianchi I model in the two schemes are analyzed. Finally, we draw the conclusions in Sec. VII.

II. CLASSICAL DYNAMICS

In this section, we review briefly the classical dynamics in Bianchi I model. As a comparison of previous investigations,we first focus on the model with a massless scalar field. Then we will turn to the case where a general matter potential is considered.

The spacetime metric of Bianchi I model is given as

$$ds^2 = -N^2 dt^2 + a_1^2 dx^2 + a_2^2 dy^2 + a_3^2 dz^2, \tag{1}$$

where N is the lapse function. When we write the classical dynamics in the Ashtekar variables, we consider the spacetime with a manifold $\Sigma \times \mathbb{R}$ where the space supersurface Σ is flat. Because of the non-compactness of the spatial manifold, it is necessary to introduce a fiducial cell $\mathcal{V}$ which has a fiducial volume $V_0 = l_1 l_2 l_3$. In Bianchi I model, the Ashtekar variables take a simple form where the phase space is given by the diagonal triad variables p_I and diagonal connection variables c_I $(I = 1, 2, 3)$. The canonical conjugate phase space satisfies

$$\{c_I, p_J\} = \kappa\gamma\delta_{IJ}, \tag{2}$$

where $\kappa = 8\pi G$ and γ is the Barbero-Immirzi parameter which was set to be $\gamma \simeq 0.2375$ by the black hole thermodynamics [13]. The triad p_I are related to the scale factors a_I by

$$|p_1| = l_2 l_3 a_2 a_3, |p_2| = l_1 l_3 a_1 a_3, |p_3| = l_1 l_2 a_1 a_2. \tag{3}$$

Thus the triad variables are the physical areas of the rectangular surface of $\mathcal{V}$ which is invariant under the coordinate rescaling. The connection variables are given by

$$c_1 = \gamma l_1 \dot{a}_1, c_2 = \gamma l_2 \dot{a}_2, c_3 = \gamma l_3 \dot{a}_3, \tag{4}$$

which is the time change rates of the physical lengths of the edges of $\mathcal{V}$, which is also invariant under the coordinate rescaling [8]. Thus, the Hamiltonian constraint in the Ashtekar variables can be written as

$$\mathcal{H}_{cl} = -\frac{N}{\kappa\gamma^2 V}\left(c_1 p_1 c_2 p_2 + c_2 p_2 c_3 p_3 + c_3 p_3 c_1 p_1\right) + \mathcal{H}_{matt},$$

where $\mathcal{H}_{matt}$ is the matter Hamiltonian. The form of the matter Hamiltonian is

$$\mathcal{H}_{matt} = N\sqrt{p_1 p_2 p_3}\rho_M.$$

Equations of motion are

$$\dot{p}_I = -\kappa\gamma\frac{\partial\mathcal{H}_{cl}}{\partial c_I}, \quad \dot{c}_I = \kappa\gamma\frac{\partial\mathcal{H}_{cl}}{\partial p_I}. \tag{5}$$

A. for a massless scalar field

In accordance with [5, 6] and [11], here we focus on a massless scalar field. For simplicity we choose the lapse function $N = \sqrt{p_1 p_2 p_3}$ and introduce a new time variable $dt' = (p_1 p_2 p_3)^{-1/2} dt$. The rescaled Hamiltonian is

$$\mathcal{H}_{cl} = -\frac{1}{\kappa\gamma^2}\left(c_1 p_1 c_2 p_2 + c_2 p_2 c_3 p_3 + c_3 p_3 c_1 p_1\right) + \frac{p_\phi^2}{2}.$$

The equations of motion are:

$$\frac{d\phi}{dt'} = p_\phi, \quad \frac{dp_\phi}{dt'} = 0, \tag{6}$$

$$\frac{dc_1}{dt'} = -\gamma^{-1} c_1 (c_2 p_2 + c_3 p_3), \tag{7}$$

$$\frac{dp_1}{dt'} = \gamma^{-1} p_1 (c_2 p_2 + c_3 p_3). \tag{8}$$

With Eq.(7) and Eq.(8) we have

$$\frac{d}{dt'}\left(c_I p_I\right) = 0 \Rightarrow c_I p_I = \kappa\gamma\hbar\mathcal{K}_I, \tag{9}$$

where $\mathcal{K}_I$ are constants. Here we define

$$p_\phi = \hbar\sqrt{\kappa}\mathcal{K}_\phi. \tag{10}$$

In addition to Eq.(9) and the Hamiltonian constraint $\mathcal{H}_{cl} = 0$ we have

$$\mathcal{K}_\phi^2 = 2(\mathcal{K}_2\mathcal{K}_3 + \mathcal{K}_3\mathcal{K}_1 + \mathcal{K}_1\mathcal{K}_2). \tag{11}$$

Combining Eq.(9) and Eq.(8) gives

$$\frac{1}{p_1}\frac{dp_1}{dt'} = \kappa\hbar\left(\mathcal{K}_2 + \mathcal{K}_3\right). \tag{12}$$

With Eq.(6) the above equation can be written to be

$$\frac{1}{p_1}\frac{dp_1}{d\phi} = \kappa\hbar\frac{\mathcal{K}_2 + \mathcal{K}_3}{p_\phi} = \sqrt{8\pi G}\left(\frac{1-\kappa_1}{\kappa_\phi}\right), \tag{13}$$

where $\mathcal{K}_I = \mathcal{K}\kappa_I$, $\mathcal{K}_\phi = \mathcal{K}\kappa_\phi$ and

$$\kappa_1 + \kappa_2 + \kappa_3 = 1, \kappa_1^2 + \kappa_2^2 + \kappa_3^2 + \kappa_\phi^2 = 1. \tag{14}$$

B. for an arbitrary matter field

Now we consider the inclusion of an arbitrary matter. Here we also choose the lapse function $N=\sqrt{p_1p_2p_3}$ and the Hamiltonian takes the form

$$\mathcal{H}_{cl}=-\frac{1}{\kappa\gamma^2}\left(c_1p_1c_2p_2+c_2p_2c_3p_3+c_3p_3c_1p_1\right)+p_1p_2p_3\rho_M.$$

The equations of motion are:

$$\frac{dp_1}{dt'}=\gamma^{-1}p_1\left(c_2p_2+c_3p_3\right), \tag{15}$$

$$\begin{aligned}\frac{dc_1}{dt'} &= -\gamma^{-1}c_1\left(c_2p_2+c_3p_3\right)\\ &\quad+\kappa\gamma p_2p_3\left(\rho_M+p_1\frac{\partial\rho_M}{\partial p_1}\right).\end{aligned} \tag{16}$$

One can see that when the energy density $\rho_M=p_\phi^2/2$ Eq.(16) turns out to be Eq.(7). Combining Eq.(15) and Eq.(16) gives the relation

$$\frac{d}{dt'}(p_Ic_I)=\kappa\gamma p_1p_2p_3\left(\rho_M+p_I\frac{\partial\rho_M}{\partial p_I}\right). \tag{17}$$

If we assume that the matter has zero anisotropy, namely $\rho_M(p_1,p_2,p_3)=\rho_M(p_1p_2p_3)$, we can get $p_I\frac{\partial\rho_M}{\partial p_I}=p_J\frac{\partial\rho_M}{\partial p_J}$, which yields

$$\frac{d}{dt'}\left(p_Ic_I-p_Jc_J\right)=0. \tag{18}$$

The above equation can be integrated to be:

$$p_Ic_I-p_Jc_J=V\left(H_I-H_J\right)=\gamma V_0\alpha_{IJ}. \tag{19}$$

with α_{IJ} being a constant anti-symmetric matrix. From the constraint $\mathcal{H}_{cl}=0$ we can get the relation

$$H_1H_2+H_2H_3+H_3H_1=\kappa\rho_M.$$

We can also define the mean scale factor a as

$$a=\left(a_1a_2a_3\right)^{1/3}, \tag{20}$$

then

$$H=\frac{\dot a}{a}=\frac{1}{3}\left(H_1+H_2+H_3\right) \tag{21}$$

and

$$\begin{aligned}H^2 &= \frac{1}{3}\left(H_1H_2+H_2H_3+H_3H_1\right)\\ &\quad+\frac{1}{18}\left[\left(H_1-H_2\right)^2+\left(H_2-H_3\right)^2+\left(H_3-H_1\right)^2\right]\\ &= \frac{\kappa}{3}\rho_M+\frac{\Sigma^2}{a^6},\end{aligned} \tag{22}$$

which is the Friedmann equation with the inclusion of anisotropy. The shear parameter is

$$\Sigma^2=\frac{1}{18}\left(\alpha_{12}^2+\alpha_{23}^2+\alpha_{31}^2\right), \tag{23}$$

which is a constant in the classical case. The anisotropic shear scalar $\sigma^2=\sigma_{\mu\nu}\sigma^{\mu\nu}$ is given by

$$\begin{aligned}\sigma^2 &= \frac{1}{3}\left[\left(H_1-H_2\right)^2+\left(H_2-H_3\right)^2+\left(H_3-H_1\right)^2\right]\\ &= \frac{6\Sigma^2}{a^6}.\end{aligned} \tag{24}$$

III. EFFECTIVE LOOP QUANTUM DYNAMICS

In the effective dynamics of LQC, the connection variables c_I $(I=1,2,3)$ should be replaced by holonomies, i.e.,

$$c_I\to\frac{\sin\left(\bar\mu_Ic_I\right)}{\bar\mu_I}, \tag{25}$$

where $\bar\mu_I$ are real valued functions of p_I which are measures of the discreteness in the quantum gravity. When $\bar\mu_I\ll1$, $\sin(\bar\mu_Ic_I)/\bar\mu_I\approx c_I$.

By choosing this, the Hamiltonian can be written as

$$\begin{aligned}\mathcal{H} &= -\frac{N}{\kappa\gamma^2V}\left[\frac{\sin\left(\bar\mu_1c_1\right)}{\bar\mu_1}\frac{\sin\left(\bar\mu_2c_2\right)}{\bar\mu_2}p_1p_2\right.\\ &\quad+\frac{\sin\left(\bar\mu_2c_2\right)}{\bar\mu_2}\frac{\sin\left(\bar\mu_3c_3\right)}{\bar\mu_3}p_2p_3\\ &\quad\left.+\frac{\sin\left(\bar\mu_3c_3\right)}{\bar\mu_3}\frac{\sin\left(\bar\mu_1c_1\right)}{\bar\mu_1}p_3p_1\right]+\mathcal{H}_{matt}.\end{aligned} \tag{26}$$

In this paper we consider the effective dynamics with higher order holonomy corrections. In fact, it is possible to approximate c_I in terms of $\sin(\bar\mu_Ic_I)$ to arbitrary accuracy

$$c_I=\frac{1}{\bar\mu}\sum_{k=0}^{\infty}\frac{(2k)!}{2^{2k}\left(k!\right)^2\left(2k+1\right)}\left[\sin\left(\bar\mu_Ic_I\right)\right]^{2k+1}. \tag{27}$$

This inspires us to define the nth order holonomized connection variable as

$$c_I^{(n)}:=\frac{1}{\bar\mu}\sum_{k=0}^{n}\frac{(2k)!}{2^{2k}\left(k!\right)^2\left(2k+1\right)}\left[\sin(\bar\mu_Ic_I)\right]^{2k+1}, \tag{28}$$

which can be made arbitrarily close to c_I but remains a periodic and bounded function of c_I. The conventional holonomy correction is corresponding to $n=0$.

With this, the Hamiltonian with holonomy corrections up to the nth order can be written as

$$\begin{aligned}\mathcal{H}_{eff} &= -\frac{N}{\kappa\gamma^2V}\left[c_1^{(n)}p_1c_2^{(n)}p_2+c_2^{(n)}p_2c_3^{(n)}p_3\right.\\ &\quad\left.+c_3^{(n)}p_3c_1^{(n)}p_1\right]+\mathcal{H}_{matt}.\end{aligned} \tag{29}$$

Using the canonical relation (2) we can get the relation

$$\left\{c_I, c_J^{(n)}\right\} = \frac{\kappa\gamma}{\bar{\mu}_J}\frac{\partial\bar{\mu}_J}{\partial p_I}\left[\cos\left(\bar{\mu}_J c_J\right)\mathfrak{S}_n\left(\bar{\mu}_J c_J\right)c_J - c_J^{(n)}\right] \tag{30}$$

$$\left\{p_I, c_J^{(n)}\right\} = -\kappa\gamma\cos(\bar{\mu}_J c_J)\mathfrak{S}_n(\bar{\mu}_J c_J)\delta_{IJ}, \tag{31}$$

where

$$\begin{aligned}\mathfrak{S}_n\left(\bar{\mu}_I c_I\right) \ : \ &= \sum_{k=0}^{n}\frac{(2k)!}{2^{2k}\left(k!\right)^2}\sin(\bar{\mu}_I c_I)^{2k} \\ &\overset{n\to\infty}{\to} \left|\cos(\bar{\mu}_I c_I)\right|^{-1}.\end{aligned} \tag{32}$$

We have to note that in Eq. (30), the equation is different for different expressions of $\bar{\mu}_I$. In the isotropic case, the parameter $\bar{\mu}$ has the form $\bar{\mu}\propto 1/\sqrt{p}$.

In Bianchi I model, the schemes are more ambiguous. Generally speaking there are two schemes that are most preferable

- $\bar{\mu}$ scheme:

$$\bar{\mu}_1 = \sqrt{\frac{\triangle}{p_1}}, \bar{\mu}_2 = \sqrt{\frac{\triangle}{p_2}}, \bar{\mu}_3 = \sqrt{\frac{\triangle}{p_3}} \tag{33}$$

- $\bar{\mu}'$ scheme:

$$\bar{\mu}_1' = \sqrt{\frac{\triangle p_1}{p_2 p_3}}, \bar{\mu}_2' = \sqrt{\frac{\triangle p_2}{p_3 p_1}}, \bar{\mu}_3' = \sqrt{\frac{\triangle p_3}{p_1 p_2}}. \tag{34}$$

Here $\triangle$ is the area gap in LQG.

IV. EFFECTIVE DYNAMICS IN $\bar{\mu}$ SCHEME

In this section, we will construct the effective dynamics in $\bar{\mu}$ scheme with higher order holonomy corrections in a massless scalar field and in an arbitrary scalar field respectively.

In $\bar{\mu}$ scheme, Eq.(30) turns out to be

$$\left\{c_I, c_J^{(n)}\right\} = -\frac{\kappa\gamma}{\bar{\mu}_J}\frac{1}{2p_J}\left[\cos\left(\bar{\mu}_J c_J\right)\mathfrak{S}_n\left(\bar{\mu}_J c_J\right)c_J - c_J^{(n)}\right]\delta_{IJ}. \tag{35}$$

In the effective dynamics we also choose the lapse function to be $N = \sqrt{p_1 p_2 p_3}$ and Eq.(29) becomes

$$\begin{aligned}\mathcal{H}_{eff} \ = \ &-\frac{1}{\kappa\gamma^2}\left[c_2^{(n)}p_2 c_3^{(n)}p_3 + c_3^{(n)}p_3 c_1^{(n)}p_1\right. \\ &\left.+c_1^{(n)}p_1 c_2^{(n)}p_2\right] + p_1 p_2 p_3 \rho_M.\end{aligned} \tag{36}$$

The equations of motion are

$$\frac{dp_1}{dt'} = \frac{1}{\gamma}\cos\left(\bar{\mu}_1 c_1\right)\mathfrak{S}_n\left(\bar{\mu}_1 c_1\right)p_1\left[c_3^{(n)}p_3 + c_2^{(n)}p_2\right], \tag{37}$$

$$\begin{aligned}\frac{dc_1}{dt'} \ = \ &-\frac{1}{\gamma}\left[\frac{3}{2}c_1^{(n)} - \frac{1}{2}\cos\left(\bar{\mu}_1 c_1\right)\mathfrak{S}_n\left(\bar{\mu}_1 c_1\right)c_1\right] \\ &\times\left[p_2 c_2^{(n)} + p_3 c_3^{(n)}\right] + \kappa\gamma p_2 p_3\left[\rho_M + p_1\frac{\partial\rho_M}{\partial p_1}\right].\end{aligned} \tag{38}$$

We also have

$$\begin{aligned}\frac{dc_1^{(n)}}{dt'} \ = \ &-\frac{1}{\gamma}c_1^{(n)}\cos\left(\bar{\mu}_1 c_1\right)\mathfrak{S}_n\left(\bar{\mu}_1 c_1\right)(c_2^{(n)}) \\ &+\kappa\gamma\mathfrak{S}_n\left(\bar{\mu}_1 c_1\right)p_2 p_3\left(\rho_M + p_1\frac{\partial\rho_M}{\partial p_1}\right).\end{aligned} \tag{39}$$

With Eq.(37) and Eq.(39) we can get

$$\begin{aligned}\frac{d}{dt'}\left(p_1 c_1^{(n)}\right) \ = \ &\kappa\gamma\mathfrak{S}_n(\bar{\mu}_1 c_1)\cos(\bar{\mu}_1 c_1) \\ &\times p_1 p_2 p_3\left(\rho_M + p_1\frac{\partial\rho_M}{\partial p_1}\right).\end{aligned} \tag{40}$$

A. for a massless scalar field

In the case of a massless scalar field, the energy density is $\rho_M = \frac{p_\phi^2}{2\sqrt{p_1 p_2 p_3}}$. We can get

$$\frac{dp_\phi}{dt'} = \{p_\phi, \mathcal{H}_{eff}\} = 0, \tag{41}$$

and

$$\frac{d\phi}{dt'} = \{\phi, \mathcal{H}_{eff}\} = p_\phi. \tag{42}$$

Eq.(41) means that p_ϕ is constant and Eq.(42) shows that ϕ can be regarded as an emergent time. With the massless scalar field Eq.(40) has the form

$$\frac{d}{dt'}\left(p_I c_I^{(n)}\right) = 0, \tag{43}$$

which means

$$p_I c_I^{(n)} = \kappa\gamma\hbar\mathcal{K}_I \tag{44}$$

where $\mathcal{K}_I$ is a constant. Comparing Eq.(44) with Eq.(9) it can be seen that the classical connection c_I is replaced by $c_I^{(n)}$. From the constraint $\mathcal{H}_{eff} = 0$ we can get the relation

$$p_\phi^2 = \frac{2}{\kappa\gamma^2}\left[c_1^{(n)}p_1 c_2^{(n)}p_2 + c_2^{(n)}p_2 c_3^{(n)}p_3 + c_3^{(n)}p_3 c_1^{(n)}p_1\right]. \tag{45}$$

As in the classical case, by defining $p_\phi = \hbar\sqrt{\kappa}\mathcal{K}_\phi$, we can get Eq.(11) and Eq.(14). Combining Eq. (37) and Eq. (44) yields

$$\frac{1}{p_1}\frac{dp_1}{dt'} = \kappa\hbar\cos(\bar{\mu}_1 c_1)\mathfrak{S}_n(\bar{\mu}_1 c_1)\left(\mathcal{K}_2 + \mathcal{K}_3\right). \tag{46}$$

By regarding ϕ as an emergent time, via Eq.(42) we can obtain

$$\frac{1}{p_1}\frac{dp_1}{d\phi} = \sqrt{\kappa\hbar}\cos(\bar{\mu}_1 c_1)\mathfrak{S}_n(\bar{\mu}_1 c_1)\frac{1-\kappa_1}{\kappa_\phi}. \tag{47}$$

When $\bar{\mu}_1 c_1 \ll 1$, $\cos(\bar{\mu}_1 c_1) \to 1$, $\sin(\bar{\mu}_1 c_1) \to 0$, $\mathfrak{S}_n \to 1$ and $c_I^{(n)} \to c_I$, Eq.(37), Eq.(38) and Eq.(39) all turn out to be the classical form. On the other hand, when $\bar{\mu}_I c_I$ is significant, the quantum corrections are more and more appreciable. When $\cos(\bar{\mu}_1 c_1) = 0$ (i.e., $\bar{\mu}_1 c_1 = \pi/2$), the quantum bounce occurs. From Eq.(28) this happens when

$$c_1^{(n)}\bar{\mu}_1 = \sum_{k=0}^{n} \frac{(2k)!}{2^{2k}\,(k!)^2\,(2k+1)} =: \mathfrak{F}_n, \tag{48}$$

where $\mathfrak{F}_n \to \pi/2$ as $n \to \infty$. From Eq.(44) and Eq.(28), we can also get

$$\begin{aligned} p_1^{3/2} &= \sqrt{\triangle}\frac{\kappa\gamma\hbar\mathcal{K}_1}{\sum_{k=0}^{n}\frac{(2k)!}{2^{2k}(k!)^2(2k+1)}\left[\sin\left(\bar{\mu}_1 c_1\right)\right]^{2k+1}} \\ &\geqslant \sqrt{\triangle}\frac{\kappa\gamma\hbar\mathcal{K}_1}{\mathfrak{F}_n}. \end{aligned} \tag{49}$$

In the second step the equality holds when $\bar{\mu}_1 c_1 = \pi/2$, which is just the bounce point. One can see that the classical singularity is never approached and the bounce is robust under the inclusion of anisotropies. We can define the directional density:

$$\rho_I := \frac{p_\phi^2}{p_I^3} \tag{50}$$

for the I-direction and its critical value is

$$\rho_{I,crit} := \frac{p_\phi^2}{p_{I,bounce}^3} = \mathfrak{F}_n^2\left(\frac{\kappa_\phi}{\kappa_I}\right)^2 \rho_{Pl}, \tag{51}$$

where $\rho_{Pl} := (\kappa\gamma^2\triangle)^{-1}$. It shows that the evolution of p_I are decoupled in three different directions. Thus the bounces occur up to three times, whenever each of the directional density reaches its critical density, which is the same as the conventional holonomy corrected case($n = 0$-order holonomy corrections). But the critical value of directional density is different from the previous case.

The mean scale factor $a(t)$ is depicted in Fig.1(a). It demonstrates that the nonsingular bouncing scenario is robust regardless of n. The quantum bounce of $a(t)$ occurs more abruptly as n increases, and if $n \to \infty$, the bounce takes place so abruptly that it only imprints a kink. The remarkable point is that even when $n \to \infty$ the effective dynamics does not reduce to the classical form.

B. for an arbitrary matter field

In this section we consider the dynamics with arbitrary matter. Here we use the method provided in [7]. We can assume that the matter density is in the form

$$\rho_M = A\left(p_1 p_2 p_3\right)^{-(1+w)/2}, \tag{52}$$

with A a constant and w the state parameter. When $a \to \infty$, the derivation in [7] has shown that the effective dynamics reduces to the classical form when $-1 < w < 1$. Although here we use the higher order holonomy corrections, this conclusion is also correct.

Then we consider the other limit $a \to 0$. When we consider the arbitrary matter, Eq.(43) is not satisfied, but we can assume that

$$p_I c_I^{(n)} = \kappa\gamma\hbar\left[\mathcal{K}_I + f_I(t)\right]. \tag{53}$$

The first term is time-independent and the second term is time-dependent. From Eq.(40) and the expression of ρ_M we can get

$$\begin{aligned} &\frac{d}{dt'}\left(p_I c_I^{(n)} - p_J c_J^{(n)}\right) \\ &= \kappa\gamma\frac{1-w}{2}A\left[\mathfrak{S}_n(\bar{\mu}_I c_I)\cos(\bar{\mu}_I c_I)\right. \\ &\quad \left.-\mathfrak{S}_n(\mu_{\bar{J}} c_J)\cos(\mu_{\bar{J}} c_J)\right]\left(p_1 p_2 p_3\right)^{\frac{1-w}{2}}. \end{aligned} \tag{54}$$

One can see when $\bar{\mu}_I c_I \to 0$ the above equation reduces to Eq.(18). When $a \to 0$, we also have $p_I \to 0$. If $w < 1$, the above equation turns out to be $\frac{d}{dt'}\left(p_I c_I^{(n)} - p_J c_J^{(n)}\right) \approx 0$, which means that $f_I(t)$ in Eq.(53) has the same value near the bouncing point: $f_1(t) = f_2(t) = f_3(t) = f(t)$. The Hamiltonian constraint $\mathcal{H}_{eff} = 0$ with $\mathcal{H}_{eff}$ given by Eq.(36) then yields

$$\begin{aligned} &3f^2(t) + 2\left(\mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3\right)f(t) \\ &+ \mathcal{K}_2\mathcal{K}_3 + \mathcal{K}_3\mathcal{K}_1 + \mathcal{K}_1\mathcal{K}_2 = \frac{A}{\kappa\hbar^2}\left(p_1 p_2 p_3\right)^{\frac{1-w}{2}}. \end{aligned} \tag{55}$$

The time-independent part satisfies

$$\mathcal{K}_2\mathcal{K}_3 + \mathcal{K}_3\mathcal{K}_1 + \mathcal{K}_1\mathcal{K}_2 = 0, \tag{56}$$

and the time-dependent part is given by:

$$\begin{aligned} f(t) &= -\frac{\mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3}{3} \\ &\quad \pm\frac{1}{3}\left[\left(\mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3\right)^2 + \frac{3A\left(p_1 p_2 p_3\right)^{\frac{1-w}{2}}}{\kappa\hbar^2}\right]^{1/2} \\ &= -\frac{\mathcal{K}}{3} \pm \frac{1}{3}\left[\mathcal{K}^2 + \frac{3A\left(p_1 p_2 p_3\right)^{\frac{1-w}{2}}}{\kappa\hbar^2}\right]^{1/2}. \end{aligned} \tag{57}$$

In the second step we scale the constants $\mathcal{K}_I = \mathcal{K}\kappa_I$ such that Eq.(56) gives

$$\kappa_1 + \kappa_2 + \kappa_3 = 1, \kappa_1^2 + \kappa_2^2 + \kappa_3^2 = 1. \tag{58}$$

In Eq.(57) we can choose the $+$ sign only without losing any generality [7]. When we discuss the bounces in the

Bianchi I model, we follow [7] to separate three cases: (i) the Kasner phase, (ii) the isotropized phase, and (iii) the transition phase.

In case (i), the contribution from the matter sector is negligible and the evolution is dominated by the constant $\mathcal{K}$. Thereby

$$\begin{aligned} f(t) &\approx -\frac{\mathcal{K}}{3}+\frac{\mathcal{K}}{3}\left[1+\frac{3A(p_1p_2p_3)^{\frac{1-w}{2}}}{8\mathcal{K}^2\kappa\hbar^2}\right] \\ &\approx \frac{A}{2\kappa\mathcal{K}\hbar^2}(p_1p_2p_3)^{\frac{1-w}{2}}. \end{aligned} \tag{59}$$

Applying it to Eq.(53) we can get

$$\begin{aligned} p_Ic_I^{(n)} &\approx \kappa\gamma\hbar\left[\mathcal{K}_I+\frac{A}{2\kappa\mathcal{K}\hbar^2}(p_1p_2p_3)^{\frac{1-w}{2}}\right] \\ &\approx \kappa\gamma\hbar\mathcal{K}_I. \end{aligned} \tag{60}$$

In the second step we use the condition that $\mathcal{K}_I \gg \frac{A}{2\kappa\mathcal{K}\hbar^2}(p_1p_2p_3)^{\frac{1-w}{2}}$. Substituting the above equation to Eq.(37) we have

$$\frac{1}{p_1}\frac{dp_1}{dt'} \approx \kappa\hbar\cos(\bar{\mu}_1c_1)\mathfrak{S}_n(\bar{\mu}_1c_1)(\mathcal{K}_2+\mathcal{K}_3).$$

In the backward evolution, the $\bar{\mu}_1c_1$ gets more and more significant, at some point $\cos(\bar{\mu}_1c_1)=0$ and the big bounce occurs. Combining Eq.(28) and Eq.(60) we can get

$$\begin{aligned} p_1^{3/2} &= \sqrt{\triangle}\frac{\kappa\gamma\hbar\mathcal{K}_1}{\sum_{k=0}^{n}\frac{(2k)!}{2^{2k}(k!)^2(2k+1)}\left[\sin\left(\bar{\mu}_1c_1\right)\right]^{2k+1}} \\ &\geqslant \sqrt{\triangle}\frac{\kappa\gamma\hbar\mathcal{K}_1}{\mathfrak{F}_n}. \end{aligned} \tag{61}$$

In the second step the equality holds at the bouncing point, which is the same as the massless scalar field. As a result the critical value of p_I is

$$p_{I,crit}=\left[\sqrt{\triangle}\frac{\kappa\gamma\hbar\mathcal{K}_1}{\mathfrak{F}_n}\right]^{2/3}. \tag{62}$$

We can also define the directional density ρ_I as

$$\rho_I := \frac{\kappa\hbar^2\mathcal{K}_I^2}{3p_I^3}, \tag{63}$$

with the expression of ρ_I we can say that the big bounces take place whenever each of the directional density reaches the critical value

$$\rho_{I,crit}=\frac{1}{3}\mathfrak{F}_n^2(\kappa\gamma^2\triangle)^{-1}\sim\rho_{pl}. \tag{64}$$

Plugging $p_{I,crit}$ to the condition $\mathcal{K}_I \gg \frac{A}{2\kappa\mathcal{K}\hbar^2}(p_1p_2p_3)^{\frac{1-w}{2}}$ it can be found that for case (i) the constant A has to satisfy

$$A \ll \mathfrak{F}_n^{1-w}\frac{|\kappa_I|\gamma^{w-1}}{|\kappa_1\kappa_2\kappa_3|^{(1-w)}}\mathcal{K}^{1+w}\kappa^w\hbar^{1+w}\triangle^{\frac{w-1}{2}}. \tag{65}$$

Now we consider case (ii) where the matter sector dominates and the universe is isotropized. In this case Eq.(57) turns out to be

$$f(t)\approx\sqrt{\frac{A}{3\kappa\hbar^2}}(p_1p_2p_3)^{\frac{1-w}{4}}. \tag{66}$$

Then Eq.(53) becomes

$$p_Ic_I^{(n)}\approx\gamma\sqrt{\frac{\kappa A}{3}}(p_1p_2p_3)^{\frac{1-w}{4}}, \tag{67}$$

where we use the condition $\mathcal{K}_I \ll \frac{A}{2\kappa\mathcal{K}\hbar^2}(p_1p_2p_3)^{\frac{1-w}{2}}$. From Eq.(37) we can get

$$\frac{1}{p_1}\frac{dp_1}{dt'}\approx 2\sqrt{\frac{\kappa A}{3}}\cos(\bar{\mu}_1c_1)\mathfrak{S}(\bar{\mu}_1c_1)(p_1p_2p_3)^{\frac{1-w}{4}}. \tag{68}$$

We can see that once again when $\cos(\bar{\mu}_1c_1)=0$, the bounce occurs. Applying Eq.(28) to Eq.(67) we have

$$p_{1,crit}^{3/2}=\frac{\gamma}{\mathfrak{F}_n}\sqrt{\frac{\kappa A\triangle}{3}}(p_{1,crit}p_2p_3)^{\frac{1-w}{4}}. \tag{69}$$

It can be seen that the critical value of p_I is coupled with other directions. If we assume in the isotropized case the bouncing points in different directions are roughly at only slightly different moments, we can get the approximated critical value for different p_I:

$$p_{crit}=\left[\frac{\kappa A\triangle\gamma^2}{3\mathfrak{F}_n^2}\right]^{\frac{2}{3+3w}}. \tag{70}$$

This means that the critical density of the bouncing point is

$$\rho_{crit}=Ap_{crit}^{-3(4+w)/2}=3\mathfrak{F}_n^2(\kappa\triangle\gamma^2)^{-1}\sim\rho_{pl}. \tag{71}$$

And in this case the criterion is

$$A\gg\gamma^{w-1}\mathcal{K}^{1+w}\kappa_I^{\frac{1+w}{2}}\hbar^{1+w}\kappa^w\triangle^{\frac{w-1}{2}}\mathfrak{F}_n^{1-w}. \tag{72}$$

Finally, in case (iii),

$$A\sim\gamma^{w-1}\mathcal{K}^{1+w}\kappa_I^{\frac{1+w}{2}}\hbar^{1+w}\kappa^w\triangle^{\frac{w-1}{2}}\mathfrak{F}_n^{1-w}, \tag{73}$$

and the bouncing points of p_I is between the critical value of p_I given in Eq.(62) and Eq.(70).

The mean scale factor $a(t)$ of the tree cases with $w=1/3$ (radiation field) in $\bar{\mu}$ scheme is depicted in Fig.1(b),(c), and (d). In each case the singularity is replaced by the big bounce regardless of the order of holonomy corrections. As the matter contribution is more and more dominant, the difference of the evolution with different holonomy orders is more and more unconspicuous.

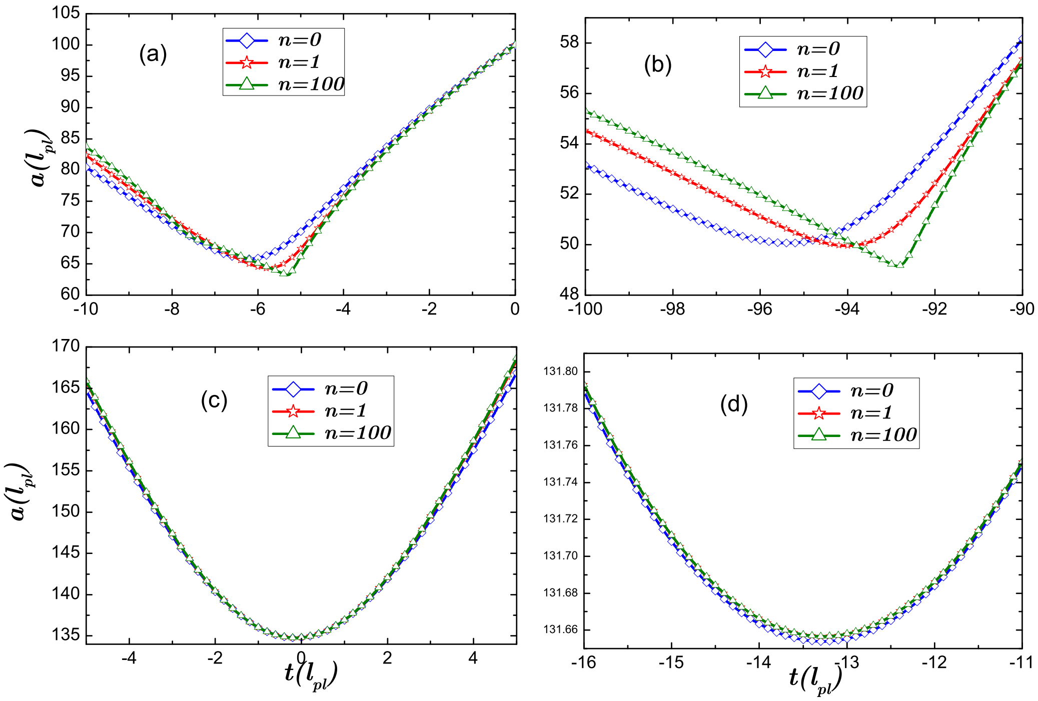

FIG. 1: Mean scale factor a(t) in $\bar{\mu}$ scheme corresponding to different orders of holonomy corrections. (a) for a massless scalar field with $\kappa_1 = -1/4$, $\kappa_2 = 3/4$, $\kappa_3 = 1/2$, and $\kappa_\phi = 1/\sqrt{8}$, $p_1(0) = p_2(0) = p_3(0) = 10^4 l_{pl}$, and $p_\phi = 2\times 10^3 \hbar\sqrt{\pi G}$. (b),(c), and (d) all for the radiation field with $w = 1/3$, $\kappa_1 = -2/7$, $\kappa_2 = 3/7$, $\kappa_3 = 6/7$, and $\mathcal{K} = 1\times 10^3$; (b) Kasner phase: $A = 0.1\hbar l_{pl}^2$, $p_1(0) = 3\times 10^4 l_{pl}$, $p_2(0) = 2\times 10^4 l_{pl}$, and $p_3(0) = 1\times 10^4 l_{pl}$. (c) Isotropized phase: $A = 10^4\hbar l_{pl}^2$, $p_1(0) = 9\times 10^4 l_{pl}$, $p_2(0) = 6\times 10^4 l_{pl}$, and $p_3(0) = 3\times 10^4 l_{pl}$. (d)Transition phase: $A = 10^2\hbar l_{pl}^2$, $p_1(0) = 3\times 10^4 l_{pl}$, $p_2(0) = 2\times 10^4 l_{pl}$, and $p_3(0) = 1\times 10^4 l_{pl}$.

V. EFFECTIVE DYNAMICS IN $\bar{\mu}'$ SCHEME

In this section we consider an alternative quantization scheme called $\bar{\mu}'$ scheme. Previous works about $\bar{\mu}'$ scheme focus on the massless scalar field as it is difficult to get the analytical solution with arbitrary matter. Here we introduce a trick named series expansion method, with which we can indeed extend the effective dynamics to arbitrary matter. As it has no difference from the arbitrary matter case, we won't discuss the massless scalar field separately.

For simplicity, we choose the lapse function $N = 1/\sqrt{p_1p_2p_3}$ and introduce a new time variable $dt'' = (p_1p_2p_3)^{1/2}dt$. We also define a new variable

$$b_I^{(n)} = \bar{\mu}_I c_I^{(n)}. \tag{74}$$

From the canonical relations we can get

$$\left\{c_I, b_J^{(n)}\right\} = \kappa\gamma\frac{\bar{\mu}'_J}{2p_I}\mathfrak{S}_n\left(\bar{\mu}'_J c_J\right)\cos\left(\bar{\mu}'_J c_J\right)c_J \quad for \quad I = J, \tag{75}$$

$$\left\{c_I, b_J^{(n)}\right\} = -\kappa\gamma\frac{\bar{\mu}'_J}{2p_I}\mathfrak{S}_n\left(\bar{\mu}'_J c_J\right)\cos\left(\bar{\mu}'_J c_J\right), \quad for \quad I \neq J \tag{76}$$

$$\left\{p_I, b_J^{(n)}\right\} = -\kappa\gamma\bar{\mu}'_J\mathfrak{S}_n\left(\bar{\mu}'_J c_J\right)\cos\left(\bar{\mu}'_J c_J\right)\delta_{IJ}. \tag{77}$$

The Hamiltonian can be written as

$$H'_{eff} = -\frac{1}{\kappa\gamma^2\triangle}\left[b_1^{(n)}b_2^{(n)} + b_2^{(n)}b_3^{(n)} + b_3^{(n)}b_1^{(n)}\right] + \rho_M. \tag{78}$$

The equations of motion are

$$\begin{aligned}\frac{dc_1}{dt''} &= -\frac{\mathfrak{S}_n(\bar{\mu}_1'c_1)\cos(\bar{\mu}_1'c_1)\bar{\mu}_1'c_1\left[b_2^{(n)}+b_3^{(n)}\right]}{2\gamma\triangle p_1}\\ &\quad+\frac{\mathfrak{S}_n(\bar{\mu}_2'c_2)\cos(\bar{\mu}_2'c_2)\bar{\mu}_2'c_2\left[b_3^{(n)}+b_1^{(n)}\right]}{2\gamma\triangle p_1}\\ &\quad+\frac{\mathfrak{S}_n(\bar{\mu}_3'c_3)\cos(\bar{\mu}_3'c_3)\bar{\mu}_3'c_3\left[b_1^{(n)}+b_2^{(n)}\right]}{2\gamma\triangle p_1}\\ &\quad+\kappa\gamma\frac{\partial\rho_M}{\partial p_1},\end{aligned} \tag{79}$$

$$\frac{dp_1}{dt''}=\frac{\mathfrak{S}_n(\bar{\mu}_1c_1)\cos(\bar{\mu}_1c_1)(b_2^{(n)}+b_3^{(n)})}{\gamma\triangle}. \tag{80}$$

Combining Eq.(79) and Eq.(80) we can get

$$\frac{d}{dt''}\left(p_Ic_I-p_Jc_J\right)=0 \tag{81}$$

As in the classical theory, $p_Ic_I-p_Jc_J$ is a constant. However, unlike the classical case Eq.(4) is no longer satisfied. From Eq.(80) we can get the relation

$$\frac{1}{p_1}\frac{dp_1}{dt''}=\frac{1}{\gamma\sqrt{\triangle p_1p_2p_3}}\mathfrak{S}_n(\bar{\mu}_1c_1)\cos(\bar{\mu}_1c_1)\bar{\mu}_1\left[b_2^{(n)}+b_3^{(n)}\right]. \tag{82}$$

It is shown that as the $\bar{\mu}$ case, when $\cos\left(\bar{\mu}_1'c_1\right)=0$ the big bounce occurs. Now we investigate the boncing regime. From Eq.(80), we can assume the relation

$$p_Ic_I=\kappa\gamma\hbar\left[\mathcal{K}_I+f(t)\right]. \tag{83}$$

Note that in this equation the time-dependent part is the same for all of p_Ic_I, which is different from the $\bar{\mu}$ case. The energy density is also assumed to be Eq.(52). Following the same derivation as in Sec.IV A we can get Eq.(56), Eq.(57) and Eq.(58). Here we also consider three cases separately.

(i) The Kasner phase:

In this phase Eq.(59) holds and we have

$$p_Ic_I\approx\kappa\gamma\hbar\mathcal{K}_I. \tag{84}$$

To get the critical condition for bouncing point, here we have to introduce a trick. With the inspiration of the method used in higher order holonomy corrections, to get the critical value of p_I we can expand the variable c_I to be

$$c_I=\frac{1}{\bar{\mu}_I'}\sum_{k=0}^{\infty}\frac{(2k)!}{2^{2k}\left(k!\right)^2(2k+1)}\left[\sin(\bar{\mu}_I'c_I)\right]^{2k+1}. \tag{85}$$

Note that this is just an algebraic series expansion which has nothing to do with the holonomy corrections. Applying the above equation to Eq.(84) we can see at the bouncing point of p_I,

$$\sqrt{p_1p_2p_3}=\frac{2}{\pi}\kappa\gamma\hbar\sqrt{\triangle}\mathcal{K}_I. \tag{86}$$

The critical energy density at this point is

$$\rho_{crit,I}=A\left[\frac{2}{\pi}\kappa\gamma\hbar\sqrt{\triangle}\mathcal{K}_I\right]^{-(1+w)}. \tag{87}$$

As in the $\bar{\mu}$ case, the criterion for the Kasner case is also $\mathcal{K}_I\gg\frac{A}{\mathcal{K}\kappa\hbar^2}\left(p_1p_2p_3\right)^{\frac{1-w}{2}}$. Applying the critical value of $\sqrt{p_1p_2p_3}$ to this condition we can get

$$A\ll(2/\pi)^{w-1}\kappa^w\gamma^{w-1}\mathcal{K}^{w+1}\kappa_I^w\hbar^{w+1}\triangle^{\frac{w-1}{2}}. \tag{88}$$

Applying it to Eq.(87) we have

$$\rho_{M,crit}\ll\left(\frac{\pi}{2}\right)^2\kappa_I^{-1}\rho_{pl}. \tag{89}$$

(ii) The isotropized phase:

In this case, Eq.(66) holds and we have

$$p_Ic_I=\gamma\sqrt{\frac{\kappa A}{3}}\left(p_1p_2p_3\right)^{\frac{1-w}{4}}. \tag{90}$$

Applying the expression of series expansion of c_I we can get

$$\sqrt{p_1p_2p_3}=\left(\frac{\pi}{2}\right)^{\frac{2}{1+w}}\left(\frac{\gamma^2\kappa A\triangle}{3}\right)^{\frac{1}{1+w}}. \tag{91}$$

You can see that for different p_I the condition at the bouncing point is the same, which means that all p_I at three directions bounce at the same time. The criterion for this case is

$$A\ll(2/\pi)^{w-1}\kappa^w\gamma^{w-1}\mathcal{K}^{w+1}\kappa_I^{(1+w)/2}\hbar^{w+1}\triangle^{\frac{w-1}{2}}. \tag{92}$$

The critical value of energy density is

$$\rho_{crit}=\left(\frac{\pi}{2}\right)^2(\gamma^2\frac{\kappa\triangle}{3})^{-1}=3\left(\frac{\pi}{2}\right)^2\rho_{Pl}\sim\rho_{pl}. \tag{93}$$

(iii) The transition phase:

Here $A\sim(2/\pi)^{w-1}\kappa^w\gamma^{w-1}\mathcal{K}^{w+1}\kappa_I^{(1+w)/2}\hbar^{w+1}\triangle^{(w-1)/2}$ and the critical value of energy density is between the one got in Eq.(89) and Eq.(93).

The numerical solutions of mean scale factor $a(t)$ is depicted in Fig.(2). The singularity is never approached and the big bounce of $a(t)$ occurs at any case. As the order of holonomy corrections increases, the big bounce takes place more and more abruptly.

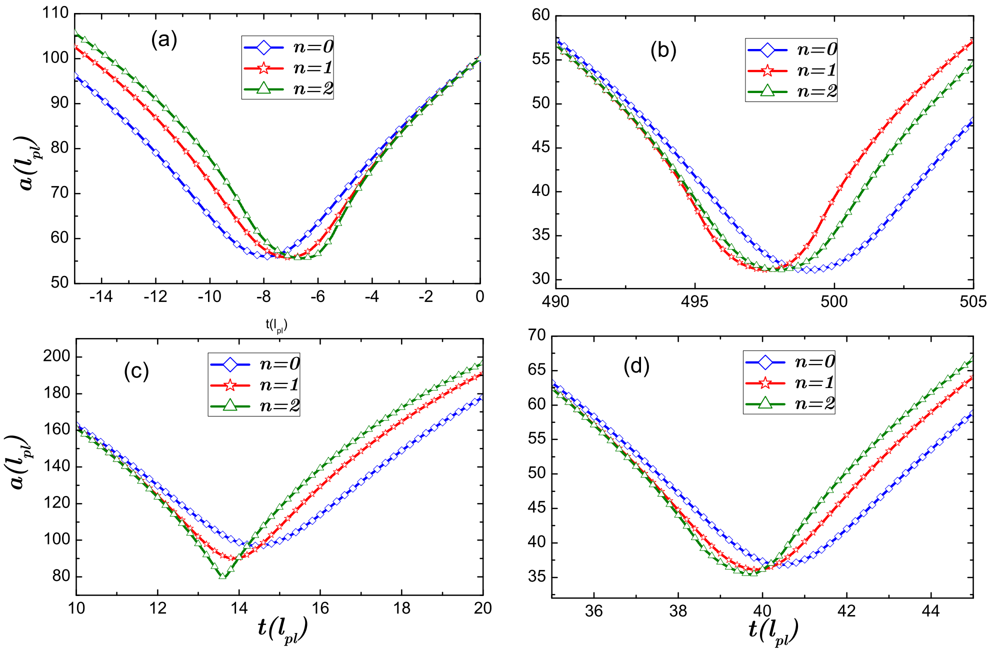

FIG. 2: Mean scale factor a(t) in $\bar{\mu}'$ scheme corresponding to different orders of holonomy corrections. (a) for aassless scalar field case with $\kappa_1 = -1/4$, $\kappa_2 = 3/4$, $\kappa_3 = 1/2$, $\kappa_\phi = 1/\sqrt{8}$, $p_1(0) = p_2(0) = p_3(0)10^4 l_{pl}$, and $p_\phi = 2 \times 10^3 \hbar\sqrt{\pi G}$. (b), (c), and (d) all for the radiation field with $w = 1/3$,$\kappa_1 = -2/7$,$\kappa_2 = 3/7$,$\kappa_3 = 6/7$;$\mathcal{K} = 1 \times 10^3$; (b)Kasner phase. With $A = 0.1\hbar l_{pl}^2$; $p_1(0) = 3 \times 10^4 l_{pl}$, $p_2(0) = 2 \times 10^4 l_{pl}$, $p_3(0) = 1 \times 10^4 l_{pl}$. (c)Isotropized phase. With$A = 10^4 \hbar l_{pl}^2$; $p_1(0) = 9 \times 10^4 l_{pl}$, $p_2(0) = 6 \times 10^4 l_{pl}$, $p_3(0) = 3 \times 10^4 l_{pl}$.(d)Transition phase. With $A = 10^2 \hbar l_{pl}^2$; and $p_1(0) = 3 \times 10^4 l_{pl}$, $p_2(0) = 2 \times 10^4 l_{pl}$, $p_3(0) = 1 \times 10^4 l_{pl}$.

VI. ANISOTROPY

In Bianchi I model the anisotropy is included. Here we use the shear parameter Σ to describe the anisotropy.

In classical case, the anisotropy is discussed in Sec. II B. We can see from Eq.(23) that in this case the shear parameter is a constant, witch does not vary when the universe evolves. However, Eq.(24) shows that when the singularity is reached, the shear scalar blows up as a approaches 0.

In the effective dynamics with higher order holonomy corrections, the shear parameter Σ is

$$\begin{aligned}\Sigma^2 &= \frac{a^6}{18}\left[(H_1-H_2)^2+(H_2-H_3)^2+(H_3-H_1)^2\right]\\ &= \frac{1}{6\gamma^2}\Big\{\Big[\cos(\bar{\mu}_2c_2)\mathfrak{S}_n(\bar{\mu}_2c_2)\left(p_1c_1^{(n)}+p_3c_3^{(n)}\right)\\ &\quad-\cos(\bar{\mu}_1c_1)\mathfrak{S}_n(\bar{\mu}_1c_1)\left(p_2c_2^{(n)}+p_3c_3^{(n)}\right)\Big]^2\\ &\quad+\Big[\cos(\bar{\mu}_3c_3)\mathfrak{S}_n(\bar{\mu}_3c_3)\left(p_1c_1^{(n)}+p_2c_2^{(n)}\right)\\ &\quad-\cos(\bar{\mu}_2c_2)\mathfrak{S}_n(\bar{\mu}_2c_2)\left(p_3c_3^{(n)}+p_1c_1^{(n)}\right)\Big]^2\\ &\quad+\Big[\cos(\bar{\mu}_1c_1)\mathfrak{S}_n(\bar{\mu}_1c_1)\left(p_2c_2^{(n)}+p_3c_3^{(n)}\right)\\ &\quad-\cos(\bar{\mu}_3c_3)\mathfrak{S}_n(\bar{\mu}_3c_3)\left(p_1c_1^{(n)}+p_2c_2^{(n)}\right)\Big]^2\Big\} \quad (94)\end{aligned}$$

Contrary to the classical case, the shear is not a constant because of the holonomy corrections. Besides that, different orders of the holonomy corrections correspond to different values of shear. In the classical regime, $\bar{\mu}_I c_I \to 0$, which means $\cos(\bar{\mu}_I c_I) = 1$, $\mathfrak{S}_n(\bar{\mu}_I c_I) = 1$ and $c_I^{(n)} \to c_I$,

the shear parameter turns out to be

$$\Sigma^2 = \frac{1}{6\gamma^2}\Big[(p_1c_1 - p_2c_2)^2 + (p_2c_2 - p_3c_3)^2 + (p_3c_3 - p_1c_1)^2\Big], \quad (95)$$

which goes back to the classical form.

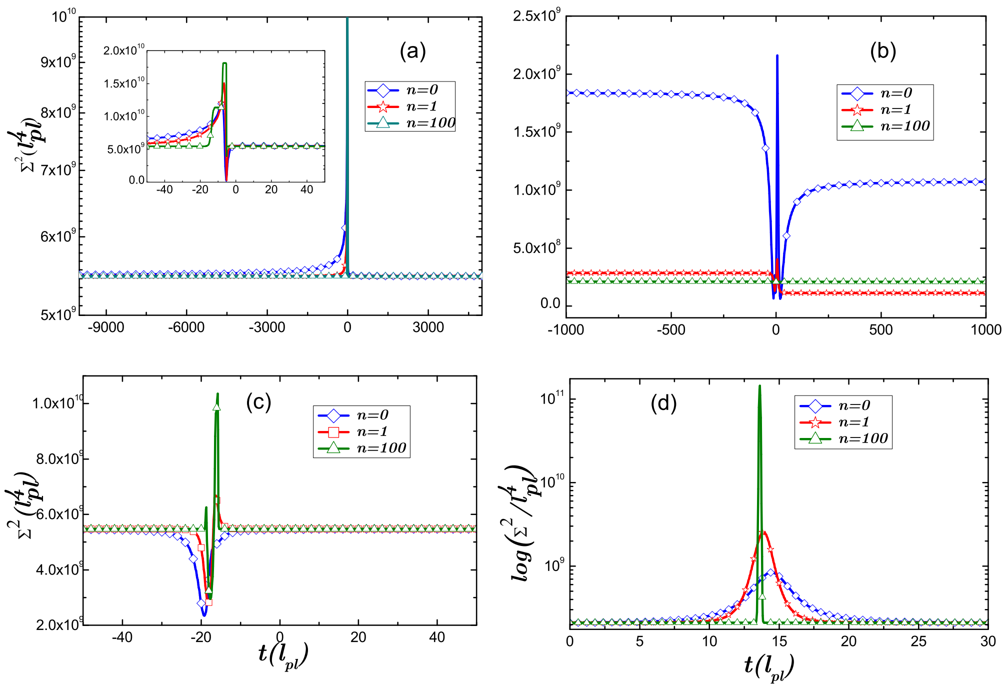

FIG. 3: Shear term in $\bar{\mu}$ and $\bar{\mu}'$ schemes for different orders of holonomy corrections. (a)Massless scalar field in $\bar{\mu}$ scheme. (b)Arbitrary matter in $\bar{\mu}$ scheme. (c)Massless scalar field in $\bar{\mu}'$ scheme. (c)Arbitrary matter in $\bar{\mu}'$ scheme.

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A. Anisotropy in $\bar{\mu}$ scheme

In $\bar{\mu}$ scheme with a massless scalar field, when we consider the pre-bounce classical regime, the shear parameter turns out to be

$$\Sigma^2 = \frac{\kappa^2\hbar^2}{6}\Big[(\mathcal{K}_1 - \mathcal{K}_2)^2 + (\mathcal{K}_2 - \mathcal{K}_3)^2 + (\mathcal{K}_3 - \mathcal{K}_1)^2\Big] \quad (96)$$

which is a constant. At the bouncing regime, although the shear parameter changes its value, the $p_I c_I^{(n)}$ keeps its constant value through the bounce. After the bounce occurs and the classical behavior is recovered, the constant $p_I c_I^{(n)}$ is the same as the pre-bounce value. So we can conclude that the shear does not change its value after the bounce. The shear for massless scalar field is depicted in Fig.3(a). One can see the shear parameter in the pre-bounce and post-bounce regime is the same. As n increases, the shear parameter also changes more abruptly.

In the Kasner phase, the shear parameter is also given by Eq.(94). In the pre-bounce classical case, the shear is in the form of Eq.(96). During the bouncing regime, although Σ varies as $\bar{\mu}_I c_I$ gets more and more significant, the $p_I c_I^{(n)} \approx \mathcal{K}_I$, which is nearly a constant. When the evolution approaches the classical regime again, its value comes back to Eq.(96). As a result, we can conclude that in the Kasner phase, we have $\Sigma^2(post\ \ bounce) \approx \Sigma^2(pre\ \ bounce)$.

Now we discuss the shear in the isotropized case. In this case, the expression is also Eq.(94). However, we can see from Eq.(67) that the $p_I c_I^{(n)} \approx$

$\gamma\sqrt{\frac{\kappa A}{3}}\left(p_1p_2p_3\right)^{(1-w)/4}$, which can not keep its value as a constant as p_I varies. So in general the shear parameter Σ is not the same in the post-bounce and pre-bounce regime as the information of anisotropy is smeared.

The shear parameter with arbitrary matter in $\bar{\mu}$ scheme is depicted in Fig.3(b). It shows that when the order of holonomy corrections increases, the variation of the shear value between post-bounce and pre-bounce regime gets more and more smaller. When $n \to \infty$, we have $\Sigma^2(post\ \ bounce) = \Sigma^2((pre\ \ bounce)$.

We have to note the case where the order of holonomy corrections $n = \infty$. In this case, $c_I^{(n)} = c_I$, $\mathfrak{S}_n(\bar{\mu}_I c_I) = 1/\left|\cos\left(\mu_{\bar{I}} c_I\right)\right|$, one may have the idea that the quantum form of shear parameter Σ turns out to be the classical form ,which may mean that the shear parameter Σ is a constant not only in the classical regime but also the bouncing regime. However, Fig.3(a) shows that it's not the case. The reason is that different p_I bounces at different moments, as a result different $\cos(\bar{\mu}_I c_I)$ flip signs at different time. From Eq.(94) one can see that if $\cos(\mu_I c_I)$ have different signs, the form of Σ can't reduce to the classical form. So even in the case where $n = \infty$ the shear parameter does vary its value at the bouncing regime, although the change is abrupt.

B. Anisotropy in $\bar{\mu}'$ Scheme

At last we discuss the shear in the $\bar{\mu}'$ scheme. The expression of the shear Σ^2 is also Eq.(94). In the classical regime, it reduces to Eq.(95). From Eq.(81) we know that $p_I c_I - p_J c_J$ is a constant all around the evolution process, including the bouncing regime. As a result, the shear is the same in the pre-bounce classical regime and the post-bounce classical regime either in the Kasner phase, transition phase or in the isotropized case.

The shear parameter with a massless scalar field and arbitrary matter is depicted in Fig.3(c) and Fig.3(d). We can see the value of pre-bounce regime is the same as post-bounce regime, regardless of the order of holonomy corrections. When $n \to \infty$, the shear does change its value in the bouncing regime as the $\bar{\mu}$ scheme.

VII. SUMMARY AND CONCLUSIONS

In this paper we investigate the Bianchi I model in LQC with higher order holonomy corrections in the heuristic effective level.

We construct an effective dynamics with higher order holonomy corrections in the forms of $\bar{\mu}$ scheme and $\bar{\mu}'$ scheme, respectively. In either case the singularity is never reached. The big bounces are robust with higher order holonomy corrections regardless of the order of the holonomy corrections, even when the order is ∞.

In the case of $\bar{\mu}$ scheme with a massless scalar field and the Kasner phase, different p_I bounces at different time, and the critical value of p_I can be found out. We use the directional density ρ_I to describe the bounces. When the directional density ρ_I reaches the critical value, the p_I bounces. In $\bar{\mu}$ scheme with isotropized phase, the three directions of p_I bounce roughly at the same time. When the energy density reaches the critical value the bounces occur at all three directions. In the case where the higher order holonomy corrections are considered, all values at the bouncing point is still bounded for all cases as the conventional holonomy corrections but timing a constant. For different orders of holonomy corrections the constants are also different but still finite.

In $\bar{\mu}'$ scheme, with a trick of series expansion we do construct the dynamics of $\bar{\mu}'$ scheme with higher order holonomy corrections for arbitrary matter. Contrary to the $\bar{\mu}$ scheme, we find that in this case the critical value of p_I is not clear but we can find out the critical energy density, and if the energy density reaches the critical value the bounce happens. In the Kasner phase different directions correspond to different critical densities while for the isotropized phase all three directions have the same critical density and the bounces happen at the same time. We also find that in the Kasner case the critical energy density is far less than the Planck density. In the isotropized case the value approaches the Planck density, which is the same as the result in [8]. Besides that, it is shown that the critical density does not depend on the order of holonomy corrections, which is different from the $\bar{\mu}$ scheme.

We use the shear parameter Σ to describe the anisotropy. In the classical case, the shear parameter is a constant. In the semiclassical regime the shear parameter is not a constant, which is a kind of quantum effects. In the $\bar{\mu}$ scheme under the condition that the holonomy correction order is finite, generically the shear parameter can not keep its constant value before and after the bounce unless at the massless scalar case. Contrary to the $\bar{\mu}$ scheme, in the $\bar{\mu}'$ scheme the shear parameter can hold its value before and after the bounce in the classical regime in all cases. When we consider the case where $n = \infty$, the shear parameter does change its value in the bouncing regime, which indicates that the variation of the shear parameter is a real quantum effect which does not depend on the artificial choice of the order of the holonomy corrections.

It's interesting when the order of holonomy corrections approaches ∞. In this case, the expression of the moving equations turns out to be the classical form, even in the bouncing regime. One may think that there is no modification and the p_I never bounces, which comes back to the classical singularity. However, it's not the case. Of course when $n = \infty$, all the equations of motion have the classical form, the dynamics is not the same. In the contracting phase before bounce, the directional Hubble parameter H_I never gets to be 0, however, at the bouncing point the H_I changes its sign abruptly and the p_I transforms from the contracting phase to the expanding phase. In other words, the Hubble parameter is discon-

tinuous at this point. This is the picture of motion in the semiclassical level, which is different from previous investigations. Surely we know that at the bounce point the evolution of the universe should be described by the rigorous quantum cosmology and the semiclassical dynamics is only an approximation, this picture is closer to the quantum picture than before.

The method of higher order holonomy corrections is a promising approach. In fact, some other problems may be solved when the order of holonomy corrections approaches ∞, i.e. the anomaly free problem in the perturbation theory of LQC [14–17]. The previous solution is to add counter terms to the Hamiltonian to eliminate the anomaly terms, but when we consider the infinite orders of holonomy corrections the anomaly terms disappear spontaneously. Whether there are other qualitative changes when all orders of holonomy corrections are considered still requires future investigations.

Now it seems that at the effective level the order of the holonomy corrections is arbitrary which can vary from 0 to ∞. However, the definite value of n is not clear. What is the definite meaning of different values of n and whether n has to approach ∞ are still questionable. This requires the rigorous quantum theory of higher order holonomy corrections. In [12] the quantum approach is constructed in homogeneous and isotropic cosmology. The construction of higher order holonomy corrections in Bianchi I model is still an open issue which is necessary to answer these questions.

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